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On Wyler's Value for the Fine-Structure Constant

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Abstract

It would appear that one of Wyler's calculations of the fine-structure constant is based on an incorrect value of the coefficient of the Poisson kernel for Cartan's third (fourth) classical domain, and hence that the value of the fine-structure constant may not be derivable from Wyler's assumptions.

An astonishingly accurate value of the fine-structure constant α was proposed by Wyler (1969, 1971) using certain geometrical arguments. There was considerable discussion (*Physics Today*, 1971; Robertson, 1971; Schwartz, 1971; Gilmore, 1972; Adler, 1973) of his work; however, neither a strong justification nor a definite disproof of his arguments appeared.

However, Wyler later issued a detailed analysis (1972) of Wyler (1969), giving a much more specific basis for his value of α. Further work, partially based on this analysis, was done by Vigier (1973), who was able to make a physical connection with quantum electrodynamics.

On the basis of Wyler's detailed analysis of his work, it would appear that Wyler's value of the fine-structure constant does not follow from his premises.

Wyler states (1972) that his value of α follows directly from the quantity

$$(2\pi)^{-5/2}[V(D^5)]^{1/4}/[V(Q^5)]^{1/2} \tag{1}$$

where $V(D^n)$ and $V(Q^n)$ are the Euclidean volumes, respectively, of Cartan's (1935; cf. Hua, 1963; Piatetsky-Chapiro, 1966) third (fourth[1]) domain and its Bergman-Silov boundary (characteristic range). Wyler (1972) further states that quantity (1) follows directly from the expression

$$P_n(z,\xi) = \frac{[V(D^n)]^{1/2}}{V(Q^n)}\frac{(1+|\tilde{z}z|^2 - 2z^\dagger z)^{n/2}}{|(\tilde{z}-\tilde{\xi})(z-\xi)|^n} \tag{2}$$

[1] Cartan (1935) refers to it as type III. Hua (1963) and Piatetsky-Chapiro (1966) refer to it as type IV.

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for the Poisson kernel for this domain, defined correctly in Wyler (1972) through

$$f(z) = \int_{Q^n} f(\xi) P_n(z, \xi)\, d\xi \tag{3}$$

where $z \in D^n$ and $\xi \in Q^n$.

Wyler's expression (2) for the Poisson kernel is unfortunately incorrect. The correct value, given in Hua (1963), does not contain the factor of $[V(D^n)]^{1/2}$, and from definition (3) the Poisson kernel is not subject to alternate normalization. The error in (2) can easily be seen for the case $n = 1$, in which case the domain is the unit disk and the boundary is the unit circle; expression (2) would give

$$P_1(r, \theta; \phi) = \frac{\pi^{1/2}}{2\pi} \frac{1 - r^2}{1 + r^2 - 2r \cos(\theta - \phi)} \tag{4}$$

which is incorrect by just the factor $\pi^{1/2} = [V(D^1)]^{1/2}$. Hence if one accepts Wyler's derivation at face value, and if all the other numerical factors in Wyler (1972) are assumed to be correct, his predicted value of the fine-structure constant would not be 1/137.036 but would be off by a factor of $[V(D^5)]^{1/4}$, which is approximately 0.6.

At this point, if one returns to Wyler (1969, 1971), one finds there that the correct value of the Poisson kernel is quoted, but the factor $[V(D^5)]^{1/4}$ is introduced as a Jacobian; its appearance in that context was considered (Robertson, 1971; Gilmore, 1972) to be a weak point in Wyler's argument.

Although I now consider Wyler's work on the fine-structure constant probably to be faulted on its own terms, it is my opinion that nevertheless he has made a contribution to physics by introducing those who studied his work to a beautiful branch of mathematics, and his very vagueness may have stimulated some hard thought.

Note Added in Proof. A further detailed discussion of Vigier (1973) recently appeared in G. B. Cvijanovich and J.-P. Vigier, *Foundations of Physics* 7, 77 (1977).

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